The Backside Parameter Viewed from Optimal Control

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Abstract

LANDING performance has long been suggested to correlate with the backside parameter, $1/T_\gamma \cong -X_u + (X_w - g/U) Z_u/Z_w$, which is a zero of the elevator to flight-path transfer function, and current flying qualities criteria require $1/T_\gamma \geqq -0.02\ \mathrm{s}^{-1}$ for the desirable flight-path stability.[1,2] Explanations concerning the physical meaning of this parameter (e.g., Refs. 2-5), however, have usually been qualitative, and this reduces the parameter's utility in quantitative discussions. For example, it has not yet been clear whether there are any differences in flying qualities between flights of a fighter and a jet transport when the respective backside parameters are the same. This paper provides an answer to this question, and shows theoretically that the parameter determines the lower bounds of the maximum achievable accuracy of the flight-path angle error due to vertical gusts, without regard to the type of airplane.

Contents

In conventional airplanes, the elevator mainly controls the flight-path angle and the throttle mainly controls the velocity. This is often called the CTOL control technique. In backside operation, whose difficulty may be hopefully related to $1/T_\gamma$, pilots must use the elevator as well as the throttle to stabilize the path control. This results in a degradation of flying qualities. The degree in which the throttle is used, however, will be strongly correlated with the airplane characteristics before the throttle loop is closed.

Assuming this, the analysis of this paper addresses the problem of flight-path angle control by the elevator alone. Also, the discussion is limited to conventional airplanes in landing approach flight conditions. The problem will first be formulated in terms of the classical Wiener filter using simplified equations. The advantage of this is that an analytical expression is possible and the role of the backside parameter is clearly disclosed. This analysis will be substantiated, using detailed equations, by a regulator analysis which is identical to the filter analysis for the single-input/single-output case.

If the pitch stabilization is tight enough, elevator control may be equivalent to specify the pitch attitude via a commanded value θ_c, and the perturbed velocity u and flight-path angle γ will be approximately given by

$$\begin{bmatrix} \dot{u} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} X_u & -UX_w \\ -Z_u/U & Z_w \end{bmatrix} \begin{bmatrix} u \\ \gamma \end{bmatrix} + \begin{bmatrix} -g+UX_w \\ -Z_w \end{bmatrix} \theta_c + \begin{bmatrix} 0 \\ -Z_w/U \end{bmatrix} w_g \tag{1}$$

where U and g denote trim velocity and acceleration due to gravity, respectively, and X_u, Z_w, etc. denote stability derivatives. Pitch angle θ_c is regarded as the control input, while w_g is the vertical gust simulated approximately as a Gaussian white noise, which will be modified later.

The pilot control is defined as θ_c, which minimizes the following criterion

$$J = \overline{\gamma^2} + \rho \cdot \overline{\theta_c^2} \tag{2}$$

where $\overline{(\ )}$ denotes mean and ρ denotes the relative weight for the control.

The block diagram for the Wiener filter formulation is shown in Fig. 1. The problem is to determine the filter transfer function $K(s)$ to minimize J, where $\gamma = \gamma_g - \gamma_f$, γ_g is the flight-path response (without control) to gust (w_g), and γ_f is the flight-path response produced by the filter output (θ_c). The transfer functions $F(s)$ and $G(s)$ can be obtained from Eq. (1) and have the following form:

$$F(s) \equiv \frac{\gamma(s)}{w_g(s)} = \frac{-Z_w}{U(s-Z_w)} \tag{3}$$

$$G(s) \equiv \frac{\gamma(s)}{\theta_c(s)} = \frac{-Z_w(s+1/T_\gamma)}{(s-X_u)(s-Z_w)} \tag{4}$$

In Eqs. (3) and (4) it is assumed $X_w = 0$ and $1/T_\gamma = -X_u - (g/U)(Z_u/Z_w)$.

The solution to this problem can be shown to result in the following optimal filter:

$$K(s) = \frac{Z_w^2}{\rho U} \frac{(-Z_w + 1/T_\gamma)(s-X_u)}{(-Z_w+\sqrt{A})(-Z_w+\sqrt{B})(s+\sqrt{A})(s+\sqrt{B})} \tag{5}$$

where

$$\sqrt{A} + \sqrt{B} = [(1+1/\rho)\cdot Z_w^2 + X_u^2 + 2(-Z_w)\sqrt{X_u^2 + (1/\rho)\cdot(1/T_\gamma)^2}\,]^{1/2} \tag{6}$$

$$\sqrt{AB} = (-Z_w)\sqrt{X_u^2 + (1/\rho)\cdot(1/T_\gamma)^2} \tag{7}$$

In the limit as $\rho \to 0$ $(\overline{\theta_c^2} \to \infty)$, $K(s)$ will tend to

$$K(s) \to \frac{1}{U} \frac{-Z_w + 1/T_\gamma}{-Z_w + |1/T_\gamma|} \cdot \frac{s - X_u}{s + |1/T_\gamma|} \tag{8}$$

Using this, the following relation between $\overline{\gamma^2}$ and $1/T_\gamma$ will be obtained, where I_{w_g} is the intensity of the Gaussian white noise, w_g.

$$\frac{\overline{\gamma^2}}{I_{w_g}} = \begin{cases} 0 & (1/T_\gamma \geq 0) \quad (9) \\ 2\left[\dfrac{-Z_w}{U(-Z_w + |1/T_\gamma|)}\right]^2 \cdot \left|\dfrac{1}{T_\gamma}\right| & (1/T_\gamma < 0) \quad (10) \end{cases}$$

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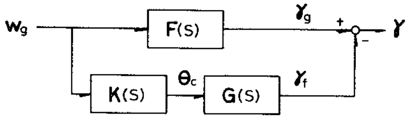

Fig. 1 Block diagram of Wiener filter analysis.

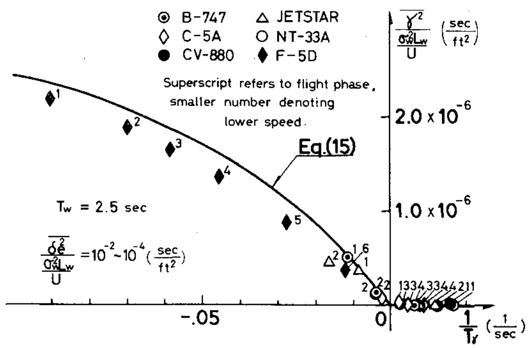

Fig. 2 Comparison between filter and regulator analyses.

Equation (10) shows that $\overline{\gamma^2}$ does not tend to zero in case of $1/T_\gamma < 0$, even if no limitations are imposed upon the control amplitudes. This is a well-known characteritic of regulators, when the system open-loop transfer function $G(s)$ has zeros with positive real parts.[6] Equation (10) also shows that $\overline{\gamma^2}$ depends strongly on $1/T_\gamma$ and the $\overline{\gamma^2}$ increases with the increase of $|1/T_\gamma|$ when $1/T_\gamma < 0$. This coincides with what has been empirically known.

Gust effects can be better simulated if w_g in Eq. (1) is replaced with w_i as

$$T_w \dot{w}_i + w_i = w_g \tag{11}$$

The intensity of w_g (previously noted as I_{w_g}) and the shaping time constant T_w are determined by the low-frequency power spectrum and the mean square gust velocity σ_w^2 of the gust model to be simulated. When the Dryden model is matched, one obtains

$$I_{w_g} = \sigma_w^2 L_w / U \tag{12}$$

$$T_w = L_w / 2U \tag{13}$$

where L_w denotes the scale length of the vertical gust.

When this gust shaping is introduced, the relation of Eqs. (9) and (10) can be shown, in the limit of $\rho \to 0$, to be modified as follows:

$$\frac{\overline{\gamma^2}}{\sigma_w^2 L_w / U} = \begin{cases} 0 & (1/T_\gamma \geq 0) \quad (14) \\ 2\left[\dfrac{-Z_w}{U(-Z_w + |1/T_\gamma|)}\right]^2 \cdot \left|\dfrac{1}{T_\gamma}\right| \cdot \left[\dfrac{1}{1+|1/T_\gamma| \cdot T_w}\right]^2 & (1/T_\gamma < 0) \quad (15) \end{cases}$$

These equations show that gust shaping reduces the $\overline{\gamma^2}$ level in proportion to the ratio of $[1/(1 + T_w \cdot |1/T_\gamma|)]^2$.

Equations (14) and (15) are plotted in Fig. 2 as a solid line, where the following numerical values are assumed: $Z_w = -0.7$ s^{-1}, $U = 200$ fps, $L_w = 1000$ ft, and $T_w = 2.5$ s.

The derivation of Eqs. (14) and (15) includes the simplifying assumption of $X_w = 0$, $\rho \to 0$ (no limitations on control magnitude), and tight pitch inner loop stabilization. In order to eliminate the associated uncertainty, the same problem was solved numerically as a stochastic regulator problem. Exact longitudinal equations of motion were used coupled with the gust dynamics [Eq. (11)]. A quadratic criterion $\overline{\gamma^2} + \rho' \cdot \overline{\delta_e^2}$ was minimized to determine optimum elevator control δ_e. The relations between $\overline{\gamma^2}$ and $1/T_\gamma$ thus obtained, are plotted also on Fig. 2 for several airplanes using symbols such as ○, ●, Δ etc., as noted on the illustration. When plotting these points, values of ρ' are selected that force $\overline{\delta_e^2}$ to be within the range of $0.1^2 \sim 0.01^2$ rad^2 when the input noise intensity $\sigma_w^2 L_w / U$ is equal to 1.0 ft^2/s. The associated $\overline{\gamma^2}$ ranges are so narrow that they are plotted as if they were single points. The stability derivatives necessary for these calculations were obtained from Refs. 7 and 8.

These regulator analysis substantiates the filter analysis for the simplified system. There is a small difference between the $\overline{\gamma^2}$ vs $1/T_\gamma$ trends in the two approaches as is seen on Fig. 2. This can be mainly attributed to the assumption that $X_w = 0$ in the filter analysis, which alters the numerical value of $1/T_\gamma$ slightly.

It should be noted from Fig. 2 that Eqs. (14) and (15) hold approximately, without regard to airplane type or flight conditions. In view of these, it will be concluded that only the backside parameter essentially determines the lower bounds of the maximum achievable flight-path accuracy in landing approach flight conditions, when the pilot behaves optimally in correcting the effects of vertical gust.

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